

(a) Let W be a walk in G of length K . Next we construct W' as follows:

If W is: $x_1 \sim x_2 \sim \dots \sim x_{k-1} \sim x_k$

then W' is: $x_1 \sim x_2 \sim \dots \sim x_{k-1} \sim x_k \sim x_{k-1}$.

Since we can repeat edges and vertices in a walk, W' is a valid walk in G of length $K+1$. Then G cannot have a longest walk.

(b) In a path we can't have repeated edges and vertices.

In a connected graph we have at least $n-1$ edges.

Thus $b = n-1$ (connected, acyclic). This means that if W is a walk of length $K > n-1$, then W is not acyclic and it must have a repeated vertex.

Thus G must have a longest path.