

a) As C_n has n vertices $\{0, 1, \dots, n-1\}$, so $C_n \times C_n$ has n^2 vertices. Namely,

$$C_n \times C_n = \{(p, q) : p \in C_n, q \in C_n\}$$

b) To show that $C_n \times C_n$ is 4-regular. For this let $n=7$, then

Let $1 \leq p \leq 5, 1 \leq q \leq 5$, then the 4 neighbours of the point (p, q) are namely,

$$(p-1, q), (p+1, q), (p, q-1), (p, q+1)$$

note that $2 \leq p+1 \leq 6$ and $2 \leq q+1 \leq 6$, and

$$0 \leq p-1 \leq 4, 0 \leq q-1 \leq 4$$

Hence each of the points are in $C_n \times C_n$. These points are shown in the graph below where the vertices of $C_7 \times C_7$ have been drawn (see the points written in red coloured pen).

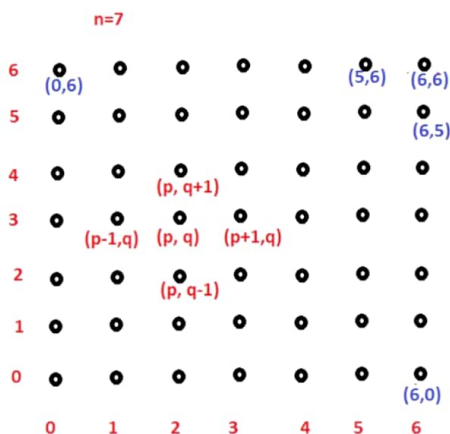
Then if a point is of the type (p, q) where p, q are either 0 or 6, then we have to use the modulo 6.

Thus in this case the 4 neighbours of the point (p, q) are namely,

$$(p-1 \pmod n, q), (p+1 \pmod n, q), (p, q-1 \pmod n), (p, q+1 \pmod n)$$

(for example see written in blue coloured pen.)

Hence $C_n \times C_n$ is 4-regular.



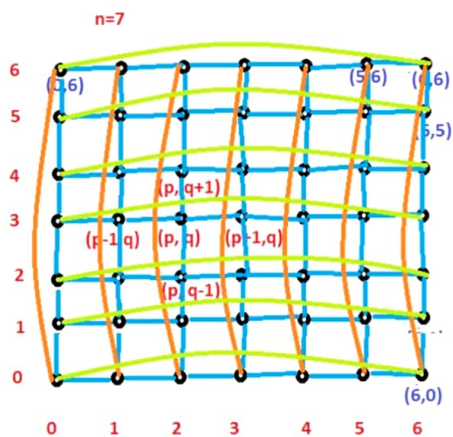
(6,6) has four neighbourhood vertices (5,6), (0,6), (6,5) and (6,0)

c) For the number of edges of $C_n \times C_n$ observe that there are n horizontal edges in each row and as there are n rows in all, so n^2 edges which are horizontal.

, and similarly, n vertical edges in each row and as there are n vertical in all, so n^2 edges which are vertical.

Thus total number of edges is $n^2 + n^2 = 2n^2$.

For example



The number of edges are $7 \times 7 + 7 \times 7 = 98$